STATE-BASED CURRICULUM REVIEW FOR MATHEMATICS

PARTICIPANT WORKBOOK
WITH SUBSTANTIATIONS

2022

Standards-in-Action 2.0

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Introduction

The SIA 2.0 State-Based Curriculum Review Tool for Mathematics is organized by four research-based dimensions for mathematics:

1. Critical Mathematical Concepts and Skills
2. Mathematical Progressions and Connections
3. Reasoning and Communicating with Mathematics
4. Quality Mathematical Tasks

The dimensions provide a set of specific criteria for both mathematical content and for supports for English learners (ELs). Each content criterion includes a set of questions designed to guide your search for evidence in the curriculum. During this training, we will focus on the dimensions sequentially, starting with Dimension 1.

The Review Process

1. After reviewing the content criteria for a dimension, search the curriculum for evidence that each criterion is met. Following each content criterion, you will find a set of "guiding questions" that are included to help you better understand the intent of the criterion.
2. Place a checkmark next to each criterion for which you found evidence. In the appropriate section, cite the location(s) in the curriculum and provide a brief comment about the evidence that substantiates your checkmark (or lack thereof).
3. Then assign the content criteria a rating of 0, 1, or 2 points for each dimension.
4. Follow the same steps to rate the EL support criteria for each dimension.

Note some content criteria are indicated with an asterisk (*). They also are research-based EL supports. When rating for EL support, therefore, consider all the EL support criteria as well as the asterisked content criteria for the dimension.

Use this workbook to analyze the extent to which the provided model curriculum aligns to the evidence-based criteria of the curriculum review tool.

Use another copy of this workbook to assess the alignment of the curriculum from your state you have chosen to review. Your team lead will maintain a master copy of your team's consensus ratings and summary findings. Once your consensus review is complete, decide, in conjunction with your state leadership, how and with whom to share the review materials.

Dimension 1
Critical Mathematical Concepts and Skills

Review Content Criteria for Dimension 1:

1 √ **Content Criterion.** Curriculum supports the development of students' understanding of the most critical mathematical concepts required by the standards for the level.

- Are most of the lessons and assessments tied to critical mathematical concepts and skills required for the level?
- Does the curriculum support students in developing conceptual understanding of those critical concepts and skills?

Substantiation:
Unit 3 addresses rates and percentages, critical concepts for Level D of the College and Career Readiness (CCR) Standards. Student understanding is supported in the lessons' Instructional Routines. For example, the Lesson 1.1 Warm Up and Launch activities encourage students to think about what they already know and work together on a task that builds their understanding of the new concept (Think-Pair-Share pp. 23–24). Teachers are also given suggestions to support students with disabilities (p. 24). These supports for the teachers translate into supports for student understanding.

2 √ **Content Criterion.** Curriculum addresses supporting concepts in ways that enhance the focus on critical concepts.

- Do activities address supporting concepts to reinforce critical concepts?
- Are supporting concepts sufficiently minimized so as not to interfere with the overall focus on the most critical concepts?

Substantiation:
All of the lessons in Unit 3 address rates and ratios, in one way or another. For example, all work with percentages is connected to rates per 100 (pp. 159, 175, 177, etc.). And in Lesson 4, the Launch problem addresses converting measurements of distance and makes a direct connection to the critical concept of rate (p. 75).

3 √ **Content Criterion.** Curriculum is designed so that students attain the required procedural skills and fluencies for the level.

- Does the curriculum provide sufficient practice opportunities for students to attain the computational and procedural fluencies for the level?
- Does the curriculum regularly assess students on the required fluencies?

Substantiation:
Warm-up and cool-down exercises are included for all lessons and are designed to strengthen procedural fluency; Practice problems for all lessons show procedural skill reinforcement (e.g., p. 202); Design Principles include emphasis on this, as well. In addition, the Unit 3 Teacher Guide, Lesson Narrative for Lessons 6.1 (p. 106) and 9.2 (p. 164) suggest work on fluency. And under Anticipated Misconceptions for Activity 9.2 (p. 136), practice with multiplication and

division computations is suggested for students not yet fluent with those operations. There are also examples of providing opportunities for students to practice calculations for fluency. For example, on page 227 in the Unit 3 Teacher Guide, we find a set of calculations with each using the number 9. In each case, the student is asked to calculate mentally and explain their calculation.

Dimension 1: Rating for Content Alignment

__√__ 2 Most or all components of the content criteria are present.

_____ 1 Some components of the content criteria are present.

_____ 0 Few or no components of the content criteria are present.

Summary Comments:
This dimension of quality (focus on the most critical content) is a particular strength of IM. Alignment with the CCR Standards ensures that the focus of the unit is on the critical concepts of the level. This unit aligns with the requirements of CCRS Level D.

Review EL Supports for Dimension 1.

1 __√__ **EL Support.** Curriculum explicitly points out the key mathematical concepts for each lesson.

- Do the student-facing materials state the concept target(s) for each lesson in clear and understandable terms?
- Are conceptual goals stated and explained for each lesson in the teacher-facing materials?

Substantiation:
The essential concepts for each lesson are clearly laid out for both the teacher and the students in Alignments, Learning Goals, and Learning Targets in the Grade 6 Course Guide. In most lessons, the introduction includes both the specific standard(s) addressed, along with standards from previous levels that are identified as "Building on" standards (e.g., Unit 3, p. 71).

2 ____ **EL Support.** Curriculum identifies key mathematical terms that students need to know to understand concepts addressed for the level.

- Are the relevant terms, required for understanding the mathematics, identified for students?
- Are teachers encouraged to prepare students for the mathematics by identifying and defining key mathematical vocabulary words?

Substantiation:
In addition to the Glossary appendices, there are notes to the teacher in the Unit 3 Teacher Guide to indicate how and when new language should be defined (e.g., Lesson 1 Narrative, p. 21). However, it would be helpful to suggest that the instructor may want to provide translations

from English of the most important glossary terms. In the Unit 3 lessons, there are several places where important terms are highlighted and defined (e.g., “unit rate” is explained in Lesson 6 beginning on p. 105). The Grade 6 Course Guide offers several suggestions for providing students with the specialized meaning of mathematical terms, including in Design Principle 1’s emphasis on “amplifying rather than simplifying speech or text” (p. 20). It is noted in the Unit 3 Teacher Guide (p. 9) that terms used in the materials that can be found in the glossary are bolded. However, the curriculum could have more explicitly identified the specific, and possibly new, math terms pertinent to each lesson in the introduction. In addition, there are a few places where vocabulary is presented but not defined. For example, in Lesson 2 of Unit 3 (p. 40), we find the terms, “length,” “volume,” “weight,” and “mass” presented in the Learning Goals. We could find neither definitions for these important terms nor an explanation of how measurements of “weight” and “mass” would differ. We also could not find evidence that students were encouraged to use the glossary, even though the Grade 6 Course Guide instructs teachers to provide a glossary of new terms for the lessons. Also noted: the glossary for Unit 3 contains 36 terms that are supposedly relevant to the unit. In this case, we found that only 10 of those terms related directly to unit rates and percentages. The majority of the terms were related to geometric figures. While we would give this criteria a partial check, it is missing some important components.

3 √ **EL Support.** Curriculum includes a glossary or encourages the use of student-friendly dictionaries that define key vocabulary so students can look up unknown words.

- Are glossary definitions written in plain language and do they include useful context or concrete examples?
- If there is no glossary, are appropriate resources suggested that can help students easily define unknown vocabulary?

Substantiation:

The Grade 6 Course Guide has an extensive, student-friendly glossary (see Appendix 3, pp. 89–106), which includes graphics, diagrams, and other visuals. A student version of the Glossary is included in the Unit 3 Teacher Guide (pp. 285–292). There are multiple instructions on how and when to use the glossary in the Scope and Sequence for each unit in the Grade 6 Course Guide (e.g., p 46). The bolding of glossary terms in the student materials adds a level of support for students to easily look up unknown words. However, as mentioned earlier, we were unable to find some key vocabulary terms in the provided glossary (e.g., weight, mass, kilogram).

4 √ **EL Support.** Curriculum uses plain language that facilitates student access to the mathematical content required in the level.

- Are the examples and explanations in the student-facing materials written concisely and without jargon?
- Do teacher-facing materials model using clear, plain language that does not distract from the mathematics?

Substantiation:

The language used in the student materials is intentionally clear, direct, active, in present or simple past tense, not overly wordy, and presented in simple sentences as described in the Grade 6 Course Guide (e.g., p. 8). In most cases, the question in a problem or task points to what is being specifically asked for. For example, Practice Problems for Chapter 3 of Lesson 1

include: “How much will he earn?”; “How many ounces of meat were used?”; or “How long will it take to …?” (pp. 34–35).

Dimension 1: Rating for EL Supports

_____ 2 Most or all components of the EL supports are present.

__√__ 1 Some components of the EL supports are present.

_____ 0 Few or no components of the EL supports are present.

Summary Comments:

Some of the recommended supports are present to support ELs’ access to critical content. However, one possible improvement would be to explicitly identify the specific, possibly new, math terms students will need to know and use in the introduction to each lesson. In addition, more emphasis for the student on how to access the meaning of new terms is needed, either in the provided glossary or in other student-friendly dictionaries.

Dimension 2
Mathematical Progressions and Connections

Review Content Criteria for Dimension 2:

1 √ **Content Criterion.** Curriculum explains ways for teachers and students to connect concepts level-to-level.

- Does the curriculum make explicit connections between the critical concepts of the level and those of future levels? For the teacher? For the student?
- Does the curriculum identify prerequisite knowledge that students will need in order to do the level-specific work?

Substantiation:

In the Unit 3 Teacher Guide, pre-unit diagnostics identify the exact standards from previous levels that are addressed (pp. 12–14). Connections are made to concepts of previous grades (e.g., Lesson 1 Standards Alignment and Lesson Narrative, pp. 21–22). References to concepts that can be connected to future learning are also included (e.g., see the Lesson Narratives for Lesson 7, p. 123, and Lesson 15, p. 251). In several Unit 3 lesson introductions, the Standards Alignment sections include standards that students will be "Building on" and "Building toward," showing connections to previous and future concepts (e.g., Lesson 3, p. 57, and Lesson 10, p. 175). In addition, the Grade 8 Course Guide includes a clear description of the geometric progressions across the levels (p. 43). On page 43 of each grade's Course Guide, we find the Unit Dependency Chart, showing connections to preceding and upcoming knowledge, as designated by standards addressed. And on page 42 of the Course Guide, there is a Curriculum Pacing Guide that compares grades 6, 7, and 8 week by week. It should be noted, however, that while the curriculum provides much information about the connections across the grades, it depends on the teacher following recommendations to make the connections apparent to the students.

2 √ **Content Criterion.** Curriculum explains ways for teachers and students to connect concepts within the level. *

- Does the curriculum clarify the connections among the critical concepts within the level? For the teacher? For the student?
- Are teachers encouraged to introduce new concepts by connecting to what students already know?
- Does content build on understanding from previous lessons, as evidenced in the table of contents or sequence of lessons?

Substantiation:

The curriculum makes connections within the level especially well for teachers. There are specific suggestions about how to make connections to previous knowledge for all students (e.g., Lesson 6, p. 106) and, more generally, for students with disabilities (e.g., p. 30 and p. 78). In the Scope and Sequence Course Overview of the Grade 6 Course Guide, connections between the units of the level are made clear (pp. 41–43). In the Overview of the Unit 3 Teacher Guide (p. 8–11), a connection is made between the previous unit (using terms such as "ratio,"

"rate," "equivalent ratios," "per," "constant speed," and "constant rate") and this unit (students represent fractional values as ratios and interpret them as rates). There are references to concepts previously learned in some Unit 3 Lesson Narratives (e.g., Lesson 1, p. 21 and Lesson 5, p. 88) and to previous units in the Grade 6 Course Guide (e.g., p. 41). And in the Unit 3 Overview, there are references to prior knowledge that will be extended in the unit (see p. 8). It is primarily left to the teacher to make these connections clear to the students.

3 √ **Content Criterion.** Curriculum devotes most attention to on-level content. Mathematics from previous levels is identified and does not interfere unduly with on-level content.

- Is mathematics content from previous levels clearly identified as "review"?
- Is the number of "review" lessons a small enough percentage of the whole so as not to distract from on-level content?
- Is the time spent reviewing past concepts purposeful and helpful, and does it support a deeper understanding of on-level content?

Substantiation:

Though, as of this writing, this curriculum is available only for grades 6–8, there are multiple references to standards from grades 4 and 5 (e.g., Lesson 1, p. 21) and even for grades 2 and 3 (e.g., Lesson 2 Standards Alignment, p. 39). The review content is appropriately placed in the introductory information, and class time used would be minimal. There are no full lessons, activities, or assessments addressing only prior-level content. In the Practice Problems at the end of each lesson, there are a few problems that relate to a previous unit or lesson. These are clearly identified for the teacher in the Unit 3 Teacher Guide (e.g., p. 29). For example, in the Lesson 4 Practice Problems, Problems 6 and 7 (p. 86) are identified as from Unit 2. Connections to previous levels are referenced only to provide context on a topic.

Dimension 2: Rating for Content Alignment

√ 2 Most or all components of the content criteria are present.

_____ 1 Some components of the content criteria are present.

_____ 0 Few or no components of the content criteria are present.

Summary Comments:

Alignment to the CCR—with emphasis on the critical areas for each level—ensures that progressions across the levels are present in the curriculum. There is a strong focus on the most critical concepts for the level, with sufficient citations for relationships to previous and future concepts. Warm-ups and Cool-downs do a good job of reviewing previous knowledge without distracting from the on-level concepts. The K–12 standards for grade 6 bridge both levels C and D in the CCR Standards. If these K–12 materials are being used for adult education classes, it would be important for teachers to pay attention to which lessons are C and which are D.

Review EL Supports for Dimension 2:

1 √ **EL Support**. Curriculum relates new mathematical vocabulary to knowledge students already have.

- When new mathematical terms are introduced, are they connected to concepts students previously learned?
- Are relationships clearly made between new vocabulary and words students may already know (e.g., "binomial" relates to "bicycle")?

Substantiation:

In the Unit 3 Teacher Guide, new concepts are related to prior knowledge, and emphasis is placed on new vocabulary (e.g., Lesson 2 Narrative, pp. 40–41). Warm-ups are designed to relate to prior knowledge. In some cases, this includes vocabulary (e.g., p. 107). Also, several Supports for Students with Disabilities include relating to previous understandings (e.g., pp. 30, 78, 92). There are opportunities throughout the materials for students to discuss and share their understanding of the concepts (e.g., Activity Syntheses pp. 25, 29, 32, 60, 66, etc.). Also, in the online student-facing materials, new glossary terms are highlighted with explanations and pictures to clarify possible misconceptions. However, in some cases the provided definitions could be more EL-friendly.

Dimension 2: Rating for EL Supports *(include asterisked Content Criterion #2 in your rating)*

√ 2 Most or all components of the EL supports are present.

_____ 1 Some components of the EL supports are present.

_____ 0 Few or no components of the EL supports are present.

Summary Comments:

The key EL support for this dimension is present. However, it should be noted that while the opportunity to use mathematical language is clearly offered, it is not always clear that the teacher would have the necessary guidance to implement effectively or to recognize the importance of that emphasis in the classroom routines.

Dimension 3
Reasoning and Communicating with Mathematics

Review Content Criteria for Dimension 3:

1 √ **Content Criterion.** Curriculum prompts students to produce mathematical arguments and to analyze the arguments of others. *

- Does the curriculum ask students to make mathematical claims and build a logical progression of statements to explore the truth of their claims?
- Are there requests in lessons, activities, and assessments for students to explain, show, or defend their findings?
- Are students asked to consider the merit of the mathematical reasoning of others?

Substantiation:

Teachers are often prompted to have students share their findings and have the class analyze different student approaches to a problem (e.g., Activity Launch and Synthesis for Activity 1.1, pp. 23–25, 29). The Grade 6 Course Guide includes general suggestions to have students critique the reasoning of others in the Design Principles (see p. 9) and in the Scope and Sequence description for Unit 3 (see p. 54). Both producing mathematical arguments and critiquing the reasoning of others are also specifically suggested throughout the Unit 3 lessons (e.g., pp. 17, 33, 78, 90, and 106). The Think-Pair-Share instructional routine is a regular way that students are prompted to present their reasoning verbally and explain themselves (e.g., pp. 23, 41, 77, 80, etc.).

2 √ **Content Criterion.** Curriculum includes sufficient supports for teachers to provide opportunities for students to build their understanding of important mathematics through discussion and verbal engagement. *

- Are there suggestions in the teacher materials for strategic grouping so that students can build and share their mathematical thinking with others?
- Do the teacher materials suggest discussion questions that will elicit student interest and response?
- Are students encouraged to verbalize their thinking and given regular opportunities to practice speaking about the mathematics they are learning?

Substantiation:

In many cases, clear instructions are given to the teacher to use group discussion to explore a concept. This is particularly the case in the Launch activities. For example, in Lesson 7.1 Launch (pp. 125–126), students are asked to think and then share with a small group to clarify their understanding of the task. And then in the second activity of the same lesson, “Think Pair Share” is suggested as an Instructional Routine (p.126). We found evidence in every lesson that students are given opportunities to verbalize about their mathematical reasoning. Support for teachers in presenting opportunities for student discussion appear in MLR8 in the Grade 6 Course Guide (see p. 8).

3 √ **Content Criterion.** Curriculum prompts teachers and students to attend to the precision of their mathematical statements. *

- Does the curriculum encourage students to be precise when they use the content-specific language they are learning, both in speaking and in writing?
- Does the curriculum encourage teachers to repeat or rephrase students' statements to model precise mathematical language and to help students clarify their thinking?
- Does the curriculum include sufficient opportunities for students to see and experience examples of precise communication in mathematics?

Substantiation:

Precise mathematical statements would necessarily use the specialized language of mathematics. Precise mathematical language is practiced and encouraged throughout the Unit 3 Teacher Guide and is a focus of the Instructional Routines suggested in the curriculum. In Lesson 1, teachers are encouraged to refrain from using or defining the term "rate per 1" until later in the unit. This is when students will be introduced to the concept of "unit rate" (p. 22). And in Lesson 10 Activity Synthesis (p. 181), the teacher is encouraged to "[f]ocus the discussion on the ways students approached the last two questions and on the precise use of language and notation." In the Grade 6 Course Guide, there is a culture of using precise mathematical language beginning with one of the second of the Task Purposes listed on page 9; in the "Why" sections of Instructional Routines (pp. 16 and 18); and in purposes of the Classroom Activities (p. 7). In the description for MLR5 in the Grade 6 Course Guide, we find that teachers are encouraged to "create space for students to produce the language of mathematical questions themselves" and to "push for clarity" (see p. 15).

4 √ **Content Criterion.** Curriculum includes examples that demonstrate mathematical reasoning and a well-structured solution, without providing formulas for solving problems. *

- Do the examples from the curriculum demonstrate reasoning for a variety of problem types and problem-solving strategies?
- Do lessons lead students to a clearer understanding of the mathematics without providing step-by-step problem-solving recipes?

Substantiation:

In the Grade 6 Course Guide, the Design Principles include several suggestions for how examples should be used in teaching and how they are used in the curriculum (e.g., pp. 8, 20, 33, 39). Principles also suggest ways to provide appropriate scaffolding (e.g., pp. 12–16). In the Algebra and Number Talk sections of the Grade 6 Course Guide, teachers are encouraged to have students share multiple strategies for solving a problem (e.g., pp. 13 and 16). In the Unit 3 materials, teachers are often steered toward providing a specific type of approach to a problem-solving situation and then show examples of how that strategy might work (e.g., pp. 18, 29, 72, and 95). And under Activity Synthesis, there are examples of focusing a class discussion on a specific strategy and "on precise use of language and notation" (e.g., p. 181). In some areas of Unit 3, there are examples of giving students a chance to think about a concept before hearing it explained. For example, in Lesson 3.1 (pp. 59–60), students are asked to measure the width of a piece of paper using Cuisenaire rods or paper clips to understand that it takes more of smaller units to find a measurement than it does for larger units.

5 √ **Content Criterion.** Curriculum promotes the strategic use of technology to support student reasoning.

- Does the curriculum encourage the use of technology as vital for mathematical curiosity and conceptual understanding?
- Does the curriculum provide opportunities and encouragement for students to use appropriate tools to assist in understanding and finding a solution to a problem?
- Are students taught when the calculator is most useful to employ and when mental calculations can be done more simply and efficiently?

Substantiation:

While four-function calculators are listed in the Required Materials lists for most lessons (e.g., p. 10.), lessons also advise the strategic use of technology. There are multiple places where strategic use of "tools" is encouraged for both students with disabilities and the general population. For example, in Lesson 1 Required Preparation (p. 22), teachers are told that even though the calculations are accessible for students at this level, using a calculator would "deemphasize computation and allow students to focus on reasoning about the context." There are a few places where the teacher is advised that calculations are accessible. There are also interactive digital simulation sites suggested for measurement (see pp. 59, 61–62). Students who need to "practice" calculations should be encouraged to work without technology (e.g., Lesson 9, p. 160).

Dimension 3: Rating for Content Alignment

√ 2 Most or all components of the content criteria are present.

_____ 1 Some components of the content criteria are present.

_____ 0 Few or no components of the content criteria are present.

Summary Comments:

These materials focus on how students express their reasoning by consistently asking students to share findings, both in writing and in discussion. Activities are structured using Five Practices for Orchestrating Productive Mathematical Discussions. This is also described in Principles to Actions: Ensuring Mathematical Success for All, as well as in Intentional Talk: How to Structure and Lead Productive Mathematical Discussions. Precise language is encouraged throughout the Grade 6 Course Guide. The Unit 3 Teacher Guide is consistent with that practice.

Review EL Supports for Dimension 3:

1 √ **EL Support.** Curriculum regularly uses mathematical language routines to allow students to strengthen their language skills while engaging in on-level mathematics as defined by the standards.

- Does the curriculum encourage teachers to promote understanding through such instructional strategies as thoughtfully sequenced questions, strategic pairing of students, and multiple reads of text?

- Are students encouraged to ask questions and share their thinking, using both written and oral language, including use of home language or translanguaging? (Translanguaging is allowing use of a multilingual person's full language repertoire rather than keeping a narrow focus on a single language.)

Substantiation:
There are many references in the materials to using Mathematical Language Routines (MLR). For example, Grade 6 Course Guide suggests using instructional routines (e.g., pp. 14–15, 22, 23, etc.), along with numerous suggestions in the Unit 3 lessons that support language understanding (e.g., Three Reads, Think-Pair-Share (e.g., pp. 23, 29, 80, 108, 151, and 279); Stronger and Clearer Each Time (e.g., pp. 95, 126, 148, and 196). Also in the Unit 3 Teacher Guide, there are frequently provided sections called Support for English Language Learners. In these, we see encouragement for teachers to provide support for strengthening use of mathematical language (e.g., p. 4). And in the Grade 6 Course Guide, there are suggestions to prompt students with "What do you notice? What do you wonder?" (see p. 15).

2 √ **EL Support.** Curriculum encourages teachers to use re-voicing to model correct mathematical language, to help students put their thoughts into words, and to clarify their responses.

- Are teachers encouraged to model accurate mathematical language for students?
- Do teacher-facing materials make specific suggestions for how students can be encouraged to precisely express their mathematical understanding verbally?

Substantiation:
Re-voicing is specifically mentioned in MLR2 and MLR5 in the Grade 6 Course Guide (p. 30). In the Unit 3 materials, there are suggestions under Support for English Language Learners, in the Activity Synthesis sections, to focus or cultivate conversations (e.g., pp. 49, 76, and 136) and to practice using mathematical language (e.g., pp. 89, 92, and 112). In some instances, the Unit 3 curriculum suggests increasing student involvement in discussion by using "Who can restate __'s reasoning in a different way?" (e.g., pp. 73, 161, and 237).

3 ___ **EL Support.** Curriculum provides acknowledgement of and ample support for students to learn specialized mathematical language, including attending to:

- Elements of mathematical words (e.g., prefixes, suffixes, and roots);
- High-value academic words (e.g., explain, interpret, etc.); and
- Multiple meanings of mathematical words (e.g., even, root, product, etc.).

Substantiation:
High-value performance verbs (e.g., analyze, explain, interpret, etc.) are used throughout the unit in both the student-facing and teacher-facing materials. They most often relate specifically to how students might approach a mathematical question and communicate their thinking. For example, "explain" is often used in student-facing materials to require student to communicate their reasoning. These appear in many different ways, including in Practice Problems (e.g., pp. 34, 54–55, and 101–102), in Launches (e.g., pp. 29, 43, 50, and 125), and in Activity and Lesson Synthesis (e.g., pp. 29, 82, and 126). The Unit 3 contexts in which students see these types of words are sufficiently rich. For example, interpretation of rates and unit rates is a theme of Unit 3. A few rich examples include unit conversion (pp. 17–18); interpretation of rational values as rates per 1 (p. 22); and applying unit rates to new situations (p. 277). In the Grade 6 Course

Guide, we find emphasis on language functions that includes support for students in understanding such high-value math terms as describe, explain, justify, generalize, critique, compare and contrast, and interpret (see pp. 30–32).

In other aspects of this criterion, we found no evidence in the curriculum of attention to the elements of words. An example where the curriculum missed an opportunity to provide this support in Unit 3 is in failing to break down the word "percent" and use its parts to help define the concept. In the Glossary, there is a definition given for "per" but it is not highlighted as a prefix. In the glossary definitions for "polyhedron/polygon" and "parallelogram" (pp. 288–289), there are similar missed opportunities.

While the glossary provides mathematical definitions in student-friendly language, language support for math terms with multiple meanings is missing. For example, some mathematical explanation of the definitions of mathematical terms, such as "face" and "net" (pp. 287–288) and "table" (p. 291) would be helpful. A partial alignment might be considered for definitions of "squared" (p. 291) and "cubed" (p. 286). However, "multiple meanings" is not really addressed, and there is more missing than present for this criterion. For this criterion, there are mixed results. However, in this case, there is more missing than present.

4 √ **EL Support.** Curriculum integrates language-based structures (e.g., linking phrases, sentence starters) to help students demonstrate their mathematical work and thinking.

- Does the curriculum offer techniques such as using sentence frames/starters, linking words, transitional phrases, and role-plays to help students express their mathematical understanding?
- Does the curriculum refer to learners' home languages as assets for learning academic content and English simultaneously?

Substantiation:

Throughout the Unit 3 materials in the Support for English Language Learners sections, there are suggestions for sentence frames and starters for teachers to suggest as they support students in explaining their own strategies (e.g., pp. 5, 66, 74, 153, etc.). In the Grade 6 Course Guide, under the MLR8 sub-section, How to Use These Materials, we find language support suggestions, including a large table of sample sentence frames for a variety of language functions (pp. 29–32).

Dimension 3: Rating for EL Supports *(include asterisked Content Criteria #1–4 in your rating)*

√ 2 Most or all components of the EL supports are present.

_____ 1 Some components of the EL supports are present.

_____ 0 Few or no components of the EL supports are present.

Summary Comments:

In this dimension, one EL support criteria cannot be fully checked. The requirements to identify, for example, "words with multiple meanings" and "elements of mathematical words" are not

specifically met in the materials. However, since extensive attention is given to the use of mathematical language, we feel this gap is not a critical loss to the overall EL supports. A few fairly simple modifications or notes to the teacher could result in a full check for most of the missing elements of the criteria. Moreover, the Unit 3 Teacher Guide has many suggestions for supporting EL students reasoning and communicating with mathematics. These include embedded supports developed by the team at Understanding Learning/Stanford Center for Assessment Learning and Equity (UL/SCALE).

Dimension 4
Quality Mathematical Tasks

Review Content Criteria for Dimension 4:

1 √ **Content Criterion.** Curriculum includes a variety of high-quality tasks that are measurable and aligned to instructional objectives outlined in the standards. *

- Are there frequent opportunities for students to engage with sufficiently challenging tasks that apply significant and engaging mathematics?
- Do tasks involve all three aspects of rigor?
- Do tasks include opportunities for students to employ relevant Standards for Mathematical Practice?
- Do tasks provide sufficient guidance for students to complete them successfully and for teachers to administer and assess them effectively?

Substantiation:
Each assessment item in Unit 3's Pre-Unit Diagnostic and End-of-Unit Assessments (pp. 12–19) is prefaced with an explanation of the task's expectation. These explanations include the level and some possible common errors. There are also tasks included within the lessons that involve parts of the modeling process (e.g., pp. 25, 27, 95) and at least one that might be presented as a full modeling task (e.g., p. 149). Also, in the lessons, there are warm-up and launch activities that include quality tasks created from situations that might occur in students' daily lives. For example, in Lesson 4 (4.4, pp. 80–81), unit conversion is presented in the context of cooking with a tablespoon. This curriculum includes many opportunities for students to apply or practice what they have learned. For example, at the end of each lesson, we find at least one practice problem (e.g., pp. 34, 52, 70, etc.) to reinforce the concepts they have learned.

2 √ **Content Criterion.** Curriculum includes application problems that are primarily on-level and that embody the critical concepts of the level.

- Are most application problems designed to promote and assess student understanding of the most critical mathematical concepts required for the level?
- Are application problems that address past or supporting concepts sufficiently minimized and clearly connected to the critical concepts for the level?

Substantiation:
Application problems in this unit all address rates, a critical concept for this level. A final optional unit for each grade is a culmination of what the students learned in the separate units and is problem based (e.g., Grade 6 Course Guide, Unit 9, Putting It All Together, pp. 71–73). The "problem-based curriculum" used in these materials ensure a strong emphasis on mathematical application. Throughout the materials, the problem-solving experiences are on level, as evidenced in standards listed in Appendix 2 of the Grade 6 Course Guide (pp. 85–88). There are no assessment items addressing future concepts.

3 √ **Content Criterion.** Curriculum suggests providing ample time for students to orient themselves to a problem and challenges them to make sense of problems without over scaffolding. *

- Do at least some problems, activities, and assessments require students to independently strategize and formulate problems and/or to defend their findings?
- Do the lessons provide examples that model reasoning for a variety of problem types, rather than providing problem-solving formulas?
- Do answer keys, rubrics, and scoring guidelines clearly connect to the requirements of targeted standards and provide sufficient guidance for interpreting student performance?

Substantiation:

The Grade 6 Course Guide Design Principles (pp. 8–10) emphasize the importance of giving all students opportunities to make sense of problems. This is further emphasized for ELs in the Grade 6 Course Guide's Mathematical Language Routines (pp. 12–13). It is also demonstrated throughout the Unit 3 Teacher Guide in the emphasis on Mathematical Practices. Many activities involve Think-Pair-Share, where students are given quiet time to think before meeting and sharing with their partner. In the Grade 6 Course Guide (e.g., p. 13), teachers are encouraged to "display one problem at a time" and to give students 1 minute of "quiet think time" for each problem. The words "quiet think time" appear 46 times in the Unit 3 materials.

The Design Principles in the Grade 6 Course Guide lay out how to provide appropriate scaffolding (e.g., pp. 8, 18, 20, etc.). The Unit 3 Teacher Guide includes much guidance about how to deliver the lessons and activities, including prompts for discussion and possible student responses. It would be important for teachers not to use the provided support suggestions to over-scaffold the work for their students. The Grade 6 Course Guide mentions the need to move students toward independent work as part of Cool Down (e.g., p. 8) and the Five Practices (e.g., p. 9).

4 √ **Content Criterion.** Curriculum correctly portrays modeling as the application of mathematics to authentic problems that arise in everyday life, society, or the workplace. Students are expected to solve un-scaffolded modeling problems that require independent thinking and decision making.

- Are there application problems that require students to do more than simply use manipulatives, explain their process, or convert between representations?
- Are there opportunities for students to solve authentic, real-world problems that require at least some modeling with mathematics?
- Are there opportunities for students to work and think independently?

Substantiation:

Modeling with mathematics is clearly addressed in the teacher support materials. Students progress from performing steps in the modeling process, with scaffolding provided by the materials and/or the teacher, to being given more autonomy. The culminating unit at the end of each grade level (for Grade 6, it is Unit 9), provides an opportunity for students to experience the full modeling experience. It is up to the teacher, however, to make sure that students have that experience, without providing the scaffolding that they may have become used to receiving. Within the lessons, the distinction between "model" and "modeling" could be clearer. For example, compare the term used in Lesson 4 Standards Alignment (pp. 71–72) with those used in Classroom Activities for Lesson 5 (p. 95) and Lesson 17 (p. 283). We also found that some

"real-life" situations were not necessarily relevant to adult students' lives (e.g., p. 96). Overall, despite these "near misses," modeling is well represented in the curriculum.

5 √ **Content Criterion.** Curriculum encourages students to produce multi-modal representations of terms and concepts when solving problems and justifying solutions. *

- Do the demands of instructional tasks and applications regularly ask students to show their thinking in multiple ways (e.g., in words and a drawing or graph).
- Do lessons regularly include examples that demonstrate multiple ways to represent mathematical concepts, both during the problem-solving process and in presenting solutions?

Substantiation:

The Unit 3 Teacher Guide includes many suggestions for teachers to encourage the use of multiple representations, which can increase engagement for students with varying learning styles and abilities. For example, in Problem 7 of the End-of-Unit Assessment (p. 18), suggestions are provided for students who "get stuck" early in their problem-solving process. They note that students who "make a table or a double number line" will probably have their efforts pay off. The use of different representations is a theme across all the Unit 3 lessons, including using different representative forms (e.g., ratios, tables, number lines, tape diagrams) and also different forms to represent quantities (e.g., fraction, decimal, or percent). In the Grade 6 Course Guide, students using various representations to explain or solve problems are an important part of the Design Principles (e.g., p. 8) and support system for students with disabilities (e.g., p. 34).

Dimension 4: Rating for Content Alignment

√ 2 Most or all components of the content criteria are present.

_____ 1 Some components of the content criteria are present.

_____ 0 Few or no components of the content criteria are present.

Summary Comments:

These materials offer a wide variety of types of tasks, including practice worksheets, assessments, word problems, real-life applications, and modeling problems. Application and modeling problems are emphasized in the Design Principles of the Grade 6 Course Guide. Quality tasks are also vital in the Unit 3 Teacher Guide's emphasis on the aspects of mathematical modeling. The need for eventual independence for all students, however, could be made clearer. This is especially true in the final problem-based unit where students apply the concepts and skills they learned in the previous units.

Review EL Supports for Dimension 4:

1 ___ **EL Support.** Curriculum encourages the teacher to present oral and visual directions, to repeat the directions, and to check for understanding by asking students to restate

them.

- Do the teacher-facing materials include suggestions for presenting directions for a task in multiple ways?
- Are teachers encouraged to check for student understanding of directions for a task or activity?

Substantiation:
We found suggestions in the Grade 6 Course Guide that students would benefit from repeated opportunities to articulate their understanding (e.g., p. 20). However, this suggestion does not pertain specifically to directions for tasks. While there is also a general suggestion that teachers "make adjustments to provide additional language support (p. 23)," it is unclear whether task directions would be one of those adjustments. There was not enough evidence found in the curriculum relating specifically to this support to warrant a check.

2 √ **EL Support.** Curriculum offers teachers guidance on how to provide explicit, constructive, and targeted feedback on student work.

- Does the curriculum include suggestions for assessing student understanding in a variety of ways?
- Are teachers encouraged to use constructive feedback for students to move them toward a better understanding of the mathematics?

Substantiation:
The MLRs in the Grade 6 Course Guide provide guidance for the in-the-moment teacher, including for self and peer assessments (p. 23). This section includes a chart that shows which MLRs support sense-making, maximize output, cultivate conversation, and maximize meta-awareness; also see How to Assess Progress in the Grade 6 Course Guide (p. 37). In some Unit 3 Anticipated Misconceptions, we find that teachers are prompted to ask struggling students to take another look at their solution and, for example, rewrite in meters (p. 31), work with fractions rather than decimals (see p. 76) or explain what a number means (see p. 110). In some cases, the Activity Synthesis also provides a narrative to support instructor feedback, including highlighting important facts and effective strategies (e.g., pp. 29, 129). Also in Unit 3, we find that teachers are instructed to "notice strategies" being used as students work so that they can have several students share approaches that were successful (e.g., p. 179). "Sample errors" are provided for assessment problems so that teachers can understand what certain errors might mean for student understanding (e.g., pp. 17–19).

Dimension 4: Rating for EL Supports *(include asterisked Content Criteria #1, #3, and #5 in your rating)*

√ 2 Most or all components of the EL supports are present.

_____ 1 Some components of the EL supports are present.

_____ 0 Few or no components of the EL supports are present.

Summary Comments:

Throughout the Grade 6 Course Guide, there are Support for English Language Learners sections that include suggestions for “Lighter Support” and “Heavier Support.” This could inspire teachers to move students from “heavy” to “light” support on their way to English language fluency. While there is no specific encouragement in the curriculum for teachers to present and repeat task directions in different ways, this could easily be remedied with a simple added instruction.

Overall Ratings and Recommendations

Determining an Overall Rating for Content Alignment: Add the total points assigned to each dimension's content criteria. The highest possible score would be 8 points (or 2 points per dimension).

Determining an Overall Rating for EL Supports: Add the total points assigned to each dimension's EL support criteria. The highest possible score would be 8 points (or 2 points per dimension).

Summarizing Comments and Recommendations: Summarize the key strengths and weaknesses of the curriculum you reviewed. Then document any recommendations regarding the continued use of the curriculum in your program(s).

Remember, it is unlikely that any curriculum will contain every content criterion and every EL support included in this review tool. Look specifically at what is missing and consider:

- How important are the missing criteria to your overall rating?
- Is their absence so critical that you cannot assign an overall rating of Well Aligned or Well Supported?
- Alternatively, how easily could you and your fellow adult educators fill in the gaps in the curriculum?

Curriculum Reviewed: Illustrative Mathematics Content Standards Level: D

Reviewer(s): SW Math Team and Pilot State Teams Date(s) of Review: May 2020

Description of the Curriculum Sample Reviewed:
Illustrative Mathematics (IM) Grade 6, Unit 3 Teacher Guide and Grade 6 Course Guide; For Dimension 2 reference: Grades 7 and 8 Course Guides.

Overall Ratings, Summary Comments, and Recommendations

Overall Rating: Content Alignment	Overall Rating: English Learner Supports
__√__ Well Aligned (6–8 points)	__√__ Well Supported (6–8 points)
_____ Somewhat Aligned (3–5 points)	_____ Somewhat Supported (3–5 points)
_____ Not Aligned (0–2 points)	_____ Not Well Supported (0–2 points)

Summary Comments and Recommendations:

The content criteria are very well addressed in this curriculum. It is particularly strong in keeping its focus on the most critical concepts of the level; demonstrating progressions within, and between, the levels; and using high-quality mathematical tasks to form a more complete understanding of the concepts, as well as student understanding. There are a few areas where English learners could be better supported. This is particularly true for mathematical language acquisition. A few fairly simple modifications or notes to the teacher could result in better alignment for most of the missing elements of the criteria.

Appendices A–D

Appendix A: Critical Concepts and Fluencies of the Level

COLOR KEY:

Black – Number
Red – Algebra
Blue – Geometry
Green – Statistics and Probability

LEVEL A

Critical Concepts of the Level
Developing understanding of whole number place value for tens and ones
Developing understanding of addition and subtraction, and the properties of these operations
Describing and reasoning about shapes and their attributes
Developing understanding of linear measurement

Fluencies of the Level
Adding and subtracting within 10

Supporting Concepts of the Level
Analyzing, comparing, creating and composing geometric shapes
Classifying and counting objects in different categories

LEVEL B

Critical Concepts of the Level
Extending understanding of base-10 notations
Adding and subtracting to 1,000; fluency and application to 100
Understanding multiplication and division of whole numbers
Understanding division as the inverse of multiplication; single-digit divisors
Developing understanding of fractions, especially unit fractions
Using standard units of measure for length, time, liquid volume and mass
Developing understanding of area and its relationship to addition and multiplication
Analyzing and partitioning two-dimensional shapes

Fluencies of the Level
Adding and subtracting within 1,000 (from memory within 100)
Multiplying and dividing within 100

Supporting Concepts of the Level
Working with equal groups of objects (foundations for multiplication)
Working with time and money
Reasoning about shapes and their attributes
Representing and interpreting data

LEVEL C

Critical Concepts of the Level

Extending the number system to positive rational numbers
Extending place value understanding for decimals to thousandths
Attaining fluency with operations, using multi-digit whole numbers and decimals
Understanding fraction equivalence and comparison
Developing fluency with sums and differences of fractions
Connecting ratio and rate to whole number multiplication and division
Writing, evaluating and interpreting expressions and equations
Developing understanding of the coordinate planes
Classifying geometric two-dimensional figures based on properties
Developing understanding of and solving problems involving volume and surface area
Developing understanding of statistical variability

Fluencies of the Level

Adding/subtracting whole numbers within 1,000,000
Performing multi-digit multiplication (from memory within 100)

Supporting Concepts of the Level

Working with factors and multiples
Converting among units within a measurement system
Representing and interpreting data

LEVEL D

Critical Concepts of the Level

Extending number sense and fluency with operations to all rational numbers
Understanding ratio and rate and using them to solve problems
Applying proportional relationships
Working with expressions and linear equations
Solving linear equations and systems of linear equations
Developing the concept of functions
Graphing functions in the coordinate plane and analyzing the graphs
Solving problems involving scale drawings
Solving problems involving two- and three-dimensional figures: area, surface area and volume
Analyzing two- and three-dimensional shapes using side length and angle measurements, similarity and congruence
Applying the Pythagorean theorem
Understanding patterns of association for bivariate data and describing them with a linear equation, when appropriate
Summarizing and interpreting data and data distributions
Understanding and applying probability concepts
Drawing inferences about populations based on random samples (probability distributions)

Fluencies of the Level

Performing multi-digit division and decimal operations
Solving linear equations of the form, px + q = r and p(x + q) = r
Solving simple 2x2 systems by inspection

Supporting Concepts of the Level

Using random sampling to draw inferences about a data population
Investigating chance processes and developing and using probability models
Investigating patterns of association in bivariate data

LEVEL E

Critical Concepts of the Level

Extending understanding of number systems to the set of real numbers
Writing equivalent expressions involving radicals and rational exponents
Reasoning quantitatively; using appropriate units and levels of precision
Defining, evaluating, comparing and modeling with linear, quadratic and exponential functions and equations
Building, interpreting and analyzing functions using different representations
Reasoning with and solving linear, quadratic and exponential equations and linear inequalities
Interpreting and using the structure of expressions to solve problems
Operating with algebraic expressions, including polynomials and rational expressions
Applying similarity and congruence concepts to geometric figures, including triangles
Using geometric models and volume formulas to solve measurement problems
Summarizing, representing and interpreting one- and two-variable data, including using frequency tables

Fluencies of the Level

Performing rational number operations
Adding, subtracting and multiplying with polynomials
Transforming expressions, using algebraic calculations (grouping, factoring, etc.)
Solving linear equations

Supporting Concepts of the Level

Understanding and applying inverse functions, including logarithmic functions
Defining trigonometric ratios on the unit circle
Modeling with trigonometric functions and their graphs
Understanding and applying conic sections

Appendix B: Critical Mathematical Concepts That Progress Across the Levels

Number and Operations:

- Level A – Base 10 addition and subtraction (2-digit); multiples of 10
- Level B – Base 10 addition and subtraction (3-digit) and multiplication (1-digit by multiples of 10); Understanding and representing fractions
- Level C – Base 10 and decimal addition, subtraction, multiplication and division (multi-digit); Operating and comparing fractions
- Level D – Rational number addition, subtraction, multiplication and division (multi-digit); Ratio and rate reasoning
- Level E – Real number addition, subtraction, multiplication and division (multi-digit)

Algebraic Thinking (Expressions, Equations and Functions):

- Level A – Using a variable to represent an unknown value
- Level B – Representing and solving problems using expressions and equations
- Level C – Writing, reading and evaluating expressions; representing and solving equations; writing an inequality
- Level D – Graphing and solving linear equations; solving pairs of simultaneous equations; understanding functions
- Level E – Interpreting expressions, equations and inequalities; understanding polynomials; solving linear, quadratic, simple rational and radical equations; using and interpreting linear, quadratic and exponential functions; graphing functions

Geometry and Measurement:

- Level A – Analyzing and composing two-dimensional shapes; understanding length measurements
- Level B – Categorizing shapes; partitioning; measuring and estimating length; measuring time and liquid volume; finding rectangular area
- Level C – Understanding and using the coordinate plane; drawing polygons; understanding angles and angle measures; measuring perimeter and area (quadrilaterals and right triangles) and volume (right rectangular prisms); converting among measurement systems
- Level D – Measuring the circumference and area of circles; understanding congruence and similarity; understanding the relationships of sides and angles in a triangle; applying the Pythagorean theorem
- Level E – Defining parallel and perpendicular; finding volumes of cylinders, cones, pyramids and spheres

Data, Statistics and Probability:

- Level A – Organizing and representing data
- Level B – Generating and graphing data
- Level C – Finding and interpreting measures of center; representing data with dot plots, box plots and histograms
- Level D – Summarizing data in a context; interpreting data using measures of center and variability; understanding and developing probability models; solving probability problems; constructing and interpreting scatterplots; displaying frequencies and relative frequencies in a two-way table
- Level E – Interpreting differences in center and spread in the context of a data set; summarizing data in a two-way frequency table; interpreting rates of change in data; distinguishing between correlation and causation

Appendix C: Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs; draw diagrams of important features and relationships; graph data and search for regularity or trends. Less experienced students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand different approaches to solving complex problems and identify correspondences between the approaches.

MP.2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: (1) the ability to decontextualize — to abstract a given situation, represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents; and (2) the ability to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, rather than just computing with them; and flexibly using different properties of operations and objects.

MP.3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They can analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them and respond to the arguments of others. They reason inductively about data, making plausible arguments that consider the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain it. Less experienced students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not initially generalized or made formal. Later, students learn to determine domains to which a mathematical assumption applies. Students at all levels can listen to or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MP.4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. This might be as simple as writing an addition expression or equation to describe a situation. A student might apply proportional reasoning to plan a school event or analyze a problem in the community. A student might use geometry to

solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically and draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are familiar with the appropriate tools for their course and are able to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various levels can identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and are able to recognize situations where working without technology is more efficient.

MP.6. Attend to precision.
Mathematically proficient students can communicate precisely to others. They use precise mathematical vocabulary in discussion with others and in their own reasoning. They understand the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the context of the problem. Less experienced students are able to articulate carefully formulated explanations to each other. By the time they reach the higher levels, they have learned to examine claims and make explicit use of mathematical definitions.

MP.7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Students in early levels, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see that 7 × 8 equals the well-remembered 7
× 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and a shift of perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP.8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look for both general methods and shortcuts. Early on, students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel out when expanding $(x–1)(x+1)$, $(x–1)(x^2+x+1)$ and $(x–1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix D: Determine if a Well-Aligned Curriculum Is Also a Good Fit for Your Program

Consider the following indicators only if your review has resulted in a rating of Well Aligned and the curriculum will support your English learners (ELs). These additional indicators will help you determine if this curriculum is a *good fit* for your program. The more that apply, the better the fit for you.

1. ___ Has evidence of positive student outcomes.

2. ___ Is affordable.

3. ___ Includes educative features and support materials for instructors.

4. ___ Reflects research and best practices.

5. ___ Is appropriate for adult education purposes.

6. ___ Appears to be relatively easy for all instructors to learn and implement.

7. ___ Includes easily adaptable lessons for all types of adult learners, including student populations with specific needs.