



# College and Career Readiness Standards-in-Action

**ADVANCED  
UNIT**

**3**

VIDEO RESOURCE PAK  
FOR MATHEMATICS

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## **OBSERVING COLLEGE AND CAREER READINESS STANDARDS-IN-ACTION**

**Video Features:**

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## INTRODUCTION

Today’s adult educators are asking important questions about college and career readiness (CCR) standards: What do lessons look like that are aligned with these standards? How should this instruction differ from instruction that was tied to previous sets of standards?

Advanced Unit 3, Observing CCR Standards-in-Action (SIA), begins to answer these questions. It offers educators the training and tools they need for observing effective practices in teaching and learning that reflect the instructional advances of CCR standards, including an examination of lesson content, instructional practices, and classroom assessments.

The video lesson of one instructor, William Walker, and the accompanying tools included in this Video Resource Pak provide additional guidance and concrete, visible evidence of CCR standards-aligned lessons. To help viewers use the CCR SIA Classroom Observation Tool, specifically, for William’s lesson, this resource includes the following elements:

- A set of guiding questions tied to each Core Action.
- A completed CCR SIA Classroom Observation Tool for Core Actions 1, 2, and 5 that identifies the steps William takes, student responses to his instruction, and standards-aligned lesson elements.\*
- William’s lesson plan, which identifies the targeted CCR standards, learning goals, and sequence of planned activities.

*\*As this is only a video segment of William’s lesson (and is not full-length), no evidence has been gathered to exemplify the indicators that make up Core Actions 3 and 4.*

### **Core Actions of the CCR SIA Classroom Observation Tool:**

**Core Action 1.** Curriculum content of the lesson matches the demands of CCR standards.

**Core Action 2.** Learning activities (questions and tasks) are challenging and maximize opportunities for students to master the lesson content.

**Core Action 3.** CCR standards are translated into lesson content that productively engages adult learners.

**Core Action 4.** The lesson is intentionally sequenced to build on and develop students’ skills and knowledge of specific content.

**Core Action 5.** Students’ levels of understanding are assessed throughout the lesson, and instruction is adjusted accordingly.



## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

### Directions:

Before watching a lesson in mathematics, review the following questions and points—then put this guide aside. Take notes on a blank classroom observation tool or separate paper. Use your notes and these questions to guide a discussion about how certain evidence determines the presence of an indicator.

Core Action 1. Curriculum content of the lesson matches the demands of the CCR standards.	
Indicator	Guiding Questions
A. Instructor presents a lesson clearly reflecting CCR content standards.	<ul style="list-style-type: none"><li>• Does the lesson reflect one or more of the CCR content standards for mathematics? When possible, refer to the lesson plan for information about the target concepts and skills of the lesson.</li><li>• Where in the lesson do students engage in a variety of tasks and activities reflective of the standards?</li></ul>
B. Instructor presents a lesson that addresses the Standards for Mathematical Practice that are central to the goals of the lesson and connected with the targeted content.	<ul style="list-style-type: none"><li>• Are any Standards for Mathematical Practice reflected in the lesson? Are they central to lesson goals?</li><li>• Do lesson activities encourage students to use these or other Standards for Mathematical Practice? Note instances and examples throughout the lesson.</li><li>• What “instructor moves” are used to bring out these habits of mind in students?</li></ul>
C. Instructor presents a lesson that matches the full depth of the requirements in the standard(s) being addressed.	<ul style="list-style-type: none"><li>• Do lesson activities address the depth of the standards being addressed?</li><li>• Are some elements of the standards not being addressed? Which ones?</li></ul>
D. Instructor establishes well-defined standard-based lesson goals.	<ul style="list-style-type: none"><li>• Are lesson goals reflected clearly in the lesson? Note instances.</li><li>• Does the instructor use the language of the standards to describe the lesson outcomes?</li><li>• Throughout the lesson, does the instructor refer to lesson objectives? What does s/he say and do?</li></ul>

## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

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<p>E. Instructor presents a lesson that focuses on the standards representing or supporting the Major Work of the Level (MWOTL).</p>	<ul style="list-style-type: none"><li>• Are the target standards of this lesson identified as MWOTL? When possible, refer to the lesson plan for evidence.</li><li>• If not, does the lesson support a standard that is MWOTL? Explain.</li></ul>
<p>F. When addressing the MWOTL, the instructor intentionally targets one or more aspects of rigor as appropriate for the addressed standard(s).</p>	<ul style="list-style-type: none"><li>• What of the following aspects of rigor does this lesson address?<ul style="list-style-type: none"><li>▪ Conceptual understanding</li><li>▪ Procedural skill and fluency</li><li>▪ Application</li></ul></li><li>• Does the instructor ask questions throughout the lesson to check student understanding of the content?</li><li>• Are students either practicing or introduced to procedural skills? If so, describe them.</li><li>• Are activities presented in a real-world context? Or is the lesson introductory so that students are not yet expected to apply their learning?</li><li>• After the instructor introduces the lesson content, can students apply the procedures to a different problem or context?</li></ul>

## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

<b>Core Action 2. Learning activities (questions and tasks) are challenging and maximize opportunities for students to master the lesson content.</b>	
<b>Indicator</b>	<b>Guiding Questions</b>
A. High-quality, challenging questions and problems prompt students to discuss their developing thoughts about the lesson content.	<ul style="list-style-type: none"> <li>What do the activities ask students to do?</li> <li>How does the instructor build challenge into the activities?</li> <li>Does the instructor ask questions to promote deeper learning?</li> </ul>
B. Students have opportunities to work with and practice level-specific problems and exercises.	<ul style="list-style-type: none"> <li>Are the activities appropriate for the level(s) of the students?</li> <li>Are there a variety of problems? Do students seem engaged?</li> </ul>
C. Instructor consistently uses explanation, modeling, or examples to make the mathematics of the lesson explicit.	<ul style="list-style-type: none"> <li>When working with students, how often does the instructor simply impart information versus ask questions, so that students will do more of the thinking and talking?</li> <li>During whole-group discussions, does the instructor use explanation, modeling, or examples, or some combination of the three?</li> </ul>
D. Instructor consistently allows appropriate wait time (3 or more seconds) after asking questions of students before prompting them for responses.	<ul style="list-style-type: none"> <li>Throughout the lesson, are students given time to express their responses and ideas?</li> <li>When asking questions, does the instructor seem comfortable with wait time for responses?</li> <li>After asking a question, does the instructor stay silent, or does s/he fill the space with more questions and explanations? Count the seconds waited at different times throughout the lesson.</li> </ul>
E. Students consistently use precise mathematics in their calculations, terminology, symbols, graphs, etc.	<ul style="list-style-type: none"> <li>Does the instructor focus on using precise mathematics with students?</li> </ul>
F. Instructor consistently asks students to elaborate on and justify their responses.	<ul style="list-style-type: none"> <li>How often does the instructor ask questions requiring elaboration or justification? Can students answer with a “yes” or “no” or a one-word answer, or must they also provide evidence and explanation?</li> <li>How often does the instructor invite students to follow up on their classmates’ comments? Note instances and examples in the lesson.</li> </ul>

## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

<b>Core Action 3. CCR Standards are translated into lesson content that productively engages adult learners.</b>	
<b>Indicator</b>	<b>Guiding Questions</b>
A. Students consistently participate actively in the lesson through class discussions and activities, group projects, etc., instead of mostly doing solitary seatwork or listening to extended lectures.	<ul style="list-style-type: none"> <li>Is the lesson dominated by instructor talk or by student talk? Estimate the minutes of each?</li> <li>Is the lesson dominated by lecture or by discussion?</li> <li>Are students engaged? What do you notice about the students: Do they seem bored or animated, or is it hard to tell?</li> <li>Do only some, many, most, or all students actively participate in activities?</li> </ul>
B. Students have varied opportunities to apply what they are learning in authentic or practical adult-oriented contexts.	<ul style="list-style-type: none"> <li>Are the activities practical and useful? Note instances and examples in the lesson.</li> <li>Are students assigned worksheets or asked to solve authentic problems?</li> <li>How can you tell that the instructor knows about the lives and interests of the students? What specific things are said to indicate that knowledge?</li> </ul>
C. When discussing or collaborating, a vast majority of students build on each other's observations or insights (e.g., showing variation in their solution methods).	<ul style="list-style-type: none"> <li>Are there opportunities for students to collaborate?</li> <li>During class discussions, does the instructor encourage students to ask each other questions or add on points?</li> </ul>
D. A vast majority of students display persistence with tasks and problems.	<ul style="list-style-type: none"> <li>When working in groups, do students seem to give up easily on a task?</li> <li>Do students appear interested in completing tasks? Do they ask questions of the instructor and their classmates to help them complete tasks?</li> </ul>

## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

Core Action 4. The lesson is intentionally sequenced to build on and develop students' skills and knowledge on specific concepts.	
Indicator	Guiding Questions
A. Instructor explicitly relates new concepts to students' knowledge.	<ul style="list-style-type: none"><li>• Does the instructor refer to previous lessons?</li></ul>
B. Instructor organizes lesson concepts in a way that build on their logical connections to each other.	<ul style="list-style-type: none"><li>• Does the instructor present the lesson so that the concepts build and flow smoothly?</li><li>• Are there other aspects of mathematics that the instructor brings seamlessly into the lesson?</li></ul>
C. Instructor makes it clear how the content of the lesson supports, and is connected to, future learning.	<ul style="list-style-type: none"><li>• Does the instructor make any connections to upcoming lessons?</li></ul>
D. Instructor ends the lesson by reviewing lesson goals; and summarizing student learning with references to student work and discussion.	<ul style="list-style-type: none"><li>• Does the instructor close the lesson?</li><li>• Does the instructor review the lesson objectives?</li><li>• What does the instructor say about the next lesson?</li></ul>

## GUIDING QUESTIONS FOR OBSERVING MATHEMATICS LESSONS

<b>Core Actions 5. Students' level of understanding is assessed during the lesson and instruction is adjusted accordingly.</b>	
<b>Indicator</b>	<b>Guiding Questions</b>
A. Instructor consistently checks for student understanding, using informal yet deliberate methods (e.g., walks around the room to check on students' work, monitors verbal responses, assigns short problems).	<ul style="list-style-type: none"> <li>When students work individually or in groups, how does the instructor check in on their understanding?</li> <li>During whole-class discussions, do you hear from many or only a handful of students?</li> </ul>
B. Instructor consistently provides students with prompt, specific feedback to correct misunderstandings, reinforce learning, and help student revise their initial work.	<ul style="list-style-type: none"> <li>What strategies does the instructor use to correct misunderstandings or reinforce learning? Note instances and examples throughout the lesson.</li> <li>When giving feedback, does the instructor use questioning to lead students to the right answers, or does s/he provide these answers to them?</li> </ul>
C. Instructor consistently provides strategic supports and scaffolds to students who need them (e.g., individualized or peer tutoring, re-teaching, review of basic skills).	<ul style="list-style-type: none"> <li>Does the instructor provide supplemental instruction? Is it needed?</li> </ul>
D. Instructor consistently provides extension activities for students who complete classwork early so they are not left idle or unchallenged.	<ul style="list-style-type: none"> <li>Does the instructor provide extension activities? Are they needed?</li> <li>Do any students seem bored while waiting for others to finish?</li> </ul>
E. A vast majority of students evaluate and reflect on their own learning.	<ul style="list-style-type: none"> <li>At the end of the lesson, does the instructor ask students to reflect on their learning—activating their metacognition? How so, is it effective?</li> </ul>

## COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

A debt of thanks is owed to the Academy of Hope Adult Public Charter School in D.C., to Mr. William Walker, and to his students for their willingness to share their teaching and learning experiences. The video lesson showcases an example of an instructor who combines sound teaching methods with CCR-aligned content, materials, and activities, as well as students who are deeply engaged in learning important concepts in mathematics. While this video lesson shows significant strength as defined by the Core Actions of the CCR SIA Classroom Observation Tool, no lesson is expected to be perfect.

## COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

Core Action 1. Curriculum content of the lesson matches the demands of CCR standards.	Y, N, or N/A
A. Instructor presents a lesson clearly reflecting CCR content standards.	Y
B. Instructor presents a lesson that addresses the Standards for Mathematical Practice that are central to the goals of the lesson and connected with the targeted content.	Y
C. Instructor presents a lesson that matches the full depth of the requirements in the standard(s) being addressed.	N
D. Instructor establishes well-defined standards-based lesson goals.	Y
E. Instructor presents a lesson that focuses on standards representing or supporting the Major Work of the Level (MWOTL).	Y
<p>F. When addressing the MWOTL, the instructor intentionally targets one or more aspects of rigor as appropriate for the addressed standard(s).</p> <p>Mark the aspect(s) of rigor the lesson addresses:</p> <ul style="list-style-type: none"> <li>• Conceptual understanding</li> <li>• Procedural skill and fluency</li> <li>• Application</li> </ul>	<p style="text-align: center;">Y</p> <ul style="list-style-type: none"> <li>• <b>Conceptual Understanding</b></li> <li>• <b>Procedural skill and fluency</b></li> </ul>
<p><b>Evidence observed:</b></p> <p><b>Indicator A:</b></p> <p>All of the CCR standards that are listed in William's lesson plan are reflected, at least partially, in his lesson:</p> <ul style="list-style-type: none"> <li>• Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (4.OA.5)</li> <li>• Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. (5.OA.2)</li> <li>• Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)</li> </ul> <p>During the lesson, students engage in a variety of tasks and activities that reflect the standards. William addresses the concept of a function as a rule that assigns each input exactly one output during the lesson. He also encourages students to identify patterns they notice in the table: For example, during the first activity, students are given a rule (two-operations) and must complete a table, with some inputs missing for corresponding outputs, and vice versa. Students report out their answers from their completed worksheet [at 00:11:39:00].</p> <p><b>Indicator B:</b></p> <p>Standards for Mathematical Practice are identified in the lesson plan and reflected in the lesson, specifically, MP.8: Look for and express regularity in repeated reasoning. To elicit these habits of mind, students must complete a function table based on a two-operation rule during the first activity. They begin to learn how to find the output based on the given rule and can use this knowledge to find the input based on a given output.</p>	

# COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

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**Evidence observed:** *Cont'd*

**Indicator C:**

Only 4.OA.5 is addressed fully in this video.

William presents a lesson that partially matches the full depth of the requirements of 8.F.1. The first part of this compound standard is central to the video segment under review. The second part of standard 8.F.1--The graph of a function is the set of ordered pairs consisting of an input and the corresponding output--is not evident during this video segment. However, graphing is likely to be part of a future lesson William only partially addresses the last part of 5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. While students record calculations with simple expressions (e.g.,  $\times 10$  and then  $+2$ ), the numbers are not in a context, making interpretation of the expressions difficult. Again, this concept would likely come later, when the students are more comfortable with finding the expressions.

**Indicator D:**

William presents the lesson goals to students at the beginning of the video segment. During this introduction, we also see interview footage of him explaining the lesson goals.

**Indicator E:**

All three standards targeted in this lesson are identified as part of the Major Work of the Level.

**Indicator F:**

Since the focus of this lesson is to introduce and explore two-operation rules for functions, conceptual understanding is the primary goal. This lesson expands on the previous lesson where students identified an input or output based on a one-operation rule. William asks questions to check student understanding. He first works through a problem with the group and then has students complete the remaining problems with a partner. He then has students report out to make sure they understand.

## COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

Core Action 2. Learning activities (questions and tasks) are challenging and maximize opportunities for students to master the lesson content.	Y, N, or N/A
A. High-quality, challenging questions and problems prompt students to discuss their developing thoughts about the lesson content.	Y
B. Students have opportunities to work with and practice level-specific problems and exercises.	Y
C. Instructor consistently uses explanation, modeling, or examples to make the mathematics of the lesson explicit.	Y
D. Instructor consistently allows appropriate wait time after asking questions of students before prompting them for responses.	Y
E. Students consistently use precise mathematics in their calculations, terminology, symbols, graphs, etc.	Y
F. Instructor consistently asks students to elaborate on and justify their responses.	Y
<p><b>Evidence observed:</b></p> <p><b>Indicator A:</b></p> <p>William makes the initial activity challenging by having students find an input from its output [at 00:02:36:00]. William shows students upfront how to find an output given the two-operation rule. He adds challenge by asking students to find the input given the output, requiring students to work backwards. Moreover, some of the partner activities include decimals, which can be challenging. After William thinks that students have a foundational understanding, he then plays a round of “the function game.” This asks students to identify another ordered pair for an unknown rule, when given a single ordered pair [at 00:13:16:00]. Students call out their guesses until several ordered pairs are written in the table on the board. Eventually all students appear to understand how the pattern works.</p> <p>Students appear to have enough knowledge to productively discuss the problems. William often asks probing questions to move students forward in their thinking. For example, when a student answers the question during the initial activity, William encourages him to elaborate on the answer [at 00:02:07:00].</p> <p><b>Indicator B:</b></p> <p>The problems and activities presented are appropriate for Low/Middle Intermediate level students and match the standards at that level. A variety of problems are presented during the short segment. William works through an example of finding the input/output, based on a two-operation rule with students, and then they complete a few problems on their own and report out. They also play “the function game” [at 00:13:16:00]. All students appear to be engaged in each of these activities and in class discussions. Students ask questions, discuss, and volunteer answers throughout the lesson.</p>	

# COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

**Core Action 2. Learning activities (questions and tasks) are challenging and maximize opportunities for students to master the lesson content.**

**Y, N, or  
N/A**

**Evidence observed:** *Cont'd*

**Indicator C:**

During the whole group discussions, William uses explanation, modeling and examples. When he introduces the concept of finding the input/output based on a two-operation rule, William builds from students' previous knowledge of doing so with one operation. After showing an example, he has students work on tables with their partners. He also asks many questions throughout.

**Indicator D:**

Throughout the lesson, William allows students plenty of time to express their answers and ideas. For example, when a student starts down the wrong path, William gives him time to express his answer before going back and addressing the misconception [at 00:04:54:00].

**Indicator E:**

William uses precise mathematical language with his students. For example, he uses the terms "input" and "output" throughout and encourages students to use precise language as well. During the beginning of the lesson, when a student answers a question using the term "operations," he praises the student for using correct terminology [at 00:01:13:00].

**Indicator F:**

William often encourages students to justify and elaborate on their responses. When students work in small and whole groups, William asks questions rather than just imparting information. His questions require more than a "yes" or "no" answer from students; he does not simply give students answers but rather leads them to the correct answers. He often tries to clarify students' understanding, asking them to explain how they got their answers. In his interview, William explains his intentionality to lead through questions [at 00:06:03:00].

**COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL  
FOR WILLIAM WALKER'S LESSON**

<b>Core Action 3. CCR standards are translated into lesson content that productively engages adult learners.</b>	<b>Y, N, or N/A</b>
A. Students consistently participate actively in the lesson through class discussions and activities, group projects, etc., instead of mostly doing solitary seatwork or listening to extended lectures.	
B. Students have varied opportunities to apply what they are learning in authentic or practical adult-oriented contexts.	
C. When discussing or collaborating, a vast majority of students build on each other's observations or insights (e.g., showing variation in their solution methods).	
D. A vast majority of students display persistence with tasks and problems.	
<p><b>Evidence observed:</b></p> <p>As this is only a video segment of William's lesson (and is not full-length), no evidence has been gathered to exemplify the indicators that make up this Core Action.</p>	

**COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL  
FOR WILLIAM WALKER'S LESSON**

<b>Core Action 4. The lesson is intentionally sequenced to build on and develop students' skills and knowledge of specific concepts.</b>	<b>Y, N, or N/A</b>
A. Instructor explicitly relates new concepts to students' knowledge.	
B. Instructor organizes lesson concepts in a way that builds on their logical connections to each other.	
C. Instructor makes it clear how the content of the lesson supports, and is connected to, future learning.	
D. Instructor ends the lesson by: <ul style="list-style-type: none"><li>• Reviewing lesson goals; and</li><li>• Summarizing student learning with references to student work and discussion.</li></ul>	
<p><b>Evidence observed:</b></p> <p>As this is only a video segment of William's lesson (and is not full-length), no evidence has been gathered to exemplify the indicators that make up this Core Action.</p>	

## COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

Core Action 5. Students' levels of understanding are assessed throughout the lesson, and instruction is adjusted accordingly.	Y, N, or N/A
A. Instructor consistently checks for student understanding, using informal yet deliberate methods (e.g., walks around the room to check on students' work, monitors verbal responses, assigns short problems).	Y
B. Instructor consistently provides students with prompt, specific feedback to correct misunderstandings, reinforce learning, and help students revise their initial work.	Y
C. Instructor consistently provides strategic supports and scaffolds to students who need them (e.g., individualized or peer tutoring, re-teaching, review of basic skills).	Y
D. Instructor consistently provides extension activities for students who complete classwork early so they are not left idle or unchallenged.	N/A
E. A vast majority of students evaluate and reflect on their own learning.	N/A
<p><b>Evidence observed:</b></p> <p><b>Indicator A:</b></p> <p>William checks for student understanding in numerous ways: Anytime students are working with a partner, he circulates, checks their work, and asks questions. After the students complete the additional problems in the initial activity, William goes over the answers to ensure students understand.</p> <p>As noted, William more often leads students to the right answers through questioning than by providing these answers to them. For example, when a student tries to check an answer using the opposite operation instead of just applying the original operation, William asks the group questions to clarify the process. He then leads the student toward correcting his answer [at 00:04:54:00]. During whole-class discussions, William calls on a several different students and students seem comfortable volunteering.</p> <p><b>Indicator B:</b></p> <p>William provides specific feedback to students throughout the lesson to correct misunderstandings. For example, when a student has trouble completing a whole group example, he first walks her through that example and then walks her through two additional examples using a series of questions [at 00:07:38:00].</p> <p><b>Indicator C:</b></p> <p>William responds during the whole group discussion to a student who has a misunderstanding when "checking" his answer [at 00:04:54:00]. He also provides supplemental instruction to a student who is having trouble working independently through the problems [at 00:07:38:00]. In both instances, he works with them until they understand the content.</p> <p><b>Indicator D:</b></p> <p>No students complete the classwork early, so extension activities are not needed.</p>	

## COMPLETED CCR SIA CLASSROOM OBSERVATION TOOL FOR WILLIAM WALKER'S LESSON

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**Evidence observed:** *Cont'd*

**Indicator E:**

Although William does not specifically ask students to reflect on their learning as a group, this video is only a segment and does not show the close of the lesson.

## LESSON PLAN FOR MATHEMATICS

*Created by William Walker • Academy of Hope Adult Public Charter School • District of Columbia*

### **1 Title of the lesson:**

Function Rules: Two-Operation Functions

### **2 Intended instructional level of the lesson:**

Low/Middle Intermediate Level

### **3 Brief description of how the lesson is to be used:**

In previous lessons, students were introduced to function rules with one operation. In this lesson, students will be introduced to function rules that involve two operations.

### **4 Learning goals of the lesson:**

Students will be able to:

1. Complete a table of values when given a two-operation function rule.
2. Identify a two-operation function rule when given a table of values.
3. Perform two-operation functions with decimals.

Functions that involve two operations give students opportunities to work with problems in authentic contexts, to brush up on arithmetic skills, and to notice patterns of change in tables of values. When completing both sides of an input/output table, students have an opportunity to “think backward,” inverting the operations used to produce an output in order to find the corresponding input.

### **5 Suggested time to spend on the lesson (e.g., number of learning sessions, hours/minutes):**

One 90-minute session

### **6 The college and career readiness standards addressing the major work of the level (MWOTL) targeted by this lesson:**

- Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (4.OA.5)  
MWOTL (Low Intermediate): Algebra – Writing, evaluating, and interpreting expressions and equations.

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. (5.OA.2)  
MWOTL (Low Intermediate): Algebra – Writing, evaluating, and interpreting expressions and equations.
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)  
MWOTL (Middle Intermediate): Functions – Developing the concept of function.  
Graphing functions in the coordinate plane and analyzing their graphs.

**7 The Standards for Mathematical Practice that are central to the goals of the lesson and their connections to the content of the lesson:**

**MP 8.** Look for and express regularity in repeated reasoning.

In this lesson, students complete several function tables based on a two-operation rule. They also identify a rule based on a table of values. Identifying patterns when completing the function tables will help them identify the rules based on a table of values.

**8 Prerequisite or foundational content students need to succeed in the lesson:**

- Students need familiarity with input/output tables.
- Students need fluency with operations and number sense.
- Students need to understand the concept of function and to know how to use one-operation rules.

**9 Description of how the content of the lesson is related to other content taught at the lesson's level:**

Previously, students were introduced to the concept of function using a function machine. Students used a one-operation rule to complete a table of values, and they were able to identify a one-operation rule that corresponds with a table of values.

In the preceding lesson, students worked with three ways to represent a function: a rule (a verbal description or equation), a table (two columns showing the corresponding “inputs” and “outputs”), and a graph (with horizontal and vertical axes to represent the inputs and outputs, respectively). After completing the current lesson, “Function Rules: Two-Operation Functions,” students will apply the content addressed in this lesson.

**10** Notes for implementing the lesson:

The suggested lesson sequence is as follows.

Show students how to “undo” the operations, working backward. When going through the example, don’t just tell students the input/output numbers; guide students in working through the problem.

Work with the whole group on the first function from the handout (shown below). It is appropriate for students to use trial and error, but ask them to make a reasoned first guess.

Rule: $x2$ then $+1$	
In	Out
2	
4	
6	
8	
	21
	31

To encourage students to think along these lines, ask them the following questions:

- Do you think the input that produces 21 will be bigger or smaller than the other inputs? Why? [Answer: Bigger, because the outputs for 2 through 8 are getting bigger]
- What is the last thing we do according to the rule? [Answer: Add 1]
- So, what was the number in this case just before we added 1? [Answer: 20]
- Good. Now, what input will get me to 20 before adding 1? [Answer: 10]

One of the most important activities in the lesson comes next, so take the necessary time for students to work through it. Before distributing the handout, ask students to figure out the two-operation rule from a table of values (like the one below). Work as a whole group on this activity

- When I go from the input to the output, how do the numbers change? [Answer: They get bigger]
- What operations often make numbers bigger? [Answer: addition and multiplication]
- What’s one way I can get from 1 to 5? [Answer: Add 4]
- Is that my function rule? Why or why not? [Answer: No, because 2 plus 4 does not equal 7]
- What else can we do to get from 1 to 5? [Answer: Multiply by 5]
- Is that our function rule? Why or why not? [Answer: No, because 2 times 5 does not equal 7]

## LESSON PLAN FOR MATHEMATICS

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- OK, it can't work with simple addition or multiplication. We are going to need to do two operations. How can we use multiplication and then addition to get from 1 to 5? [*Answer: Multiply by 4 and then add 1*]
- Does that work for the rest of the inputs/outputs? Why or why not? [*Answer: No—it doesn't work for 2 ( $2 \times 4 = 8$ ,  $8 + 1 = 9$ ; I want 2 to produce 7)*]

[*Solution: input  $\times 2$  and then  $+ 3 =$  output, or times 2 and then add 3]*

Reminders:

- It is good for students to spend some time using trial and error to figure out the function rules. Don't jump in to offer shortcuts.
- Be on the lookout for tables that can be described by more than one rule.
- If students happen to notice there is a constant change in the outputs when the inputs are consecutive, praise them and ask them if they see a pattern. This will be explored more deeply in subsequent lessons.

Now it is time to play the “function game”. Create a one- or two-operation function rule using multiplication and/or addition (like the solution for the table below). Complete a few inputs and outputs. Ask students to offer an input and output based on what they think the rule is. Tell students if their pairs are correct or not but do not tell them the rule. The goal is for students to guess the rule.

<b>Rule:</b>	
In	Out
5	12
6	14

[*Solution: input  $\times 2$  and then  $+ 2 =$  output, or multiply by 2 and then add 2]*